references

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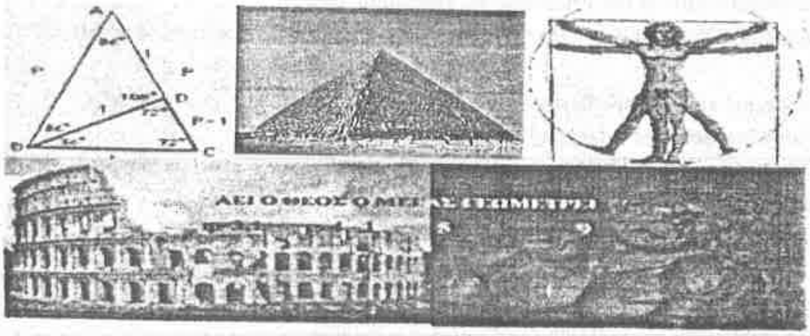
Since extensive written information concerning the mathematical background of the Ancient Egyptians is missing, an effort was made to use their impressive constructions, namely the pyramids, to complete our knowledge and understanding of that period.

Measurements were performed, which obviously involved uncertainties due to the devices used and the fact that the dimensions measured were affected by geological and physical phenomena.

Based on the measurements of the Great Pyramid made by Sir Petrie, we can conclude that, within experimental error, the geometry of the pyramid encodes at least two mathematical constants: π (pi, the ratio of the circumference to the diameter of a circle) and Φ (the golden section).



The vital questions are: "did they know?" and "how did they arrive in the correct expressions concerning the volume of truncated pyramids?". There are indications that they might knew, or at least, that they had the feeling of it. Further research is obviously needed, which has to be based on scientific arguments and facts and not on subjective interpretations, although the latter is very tempting.

In any case one thing is certain: Arabic mathematical knowledge transferred through the centuries, greatly influenced other civilizations and played an important role in the development of the Western culture as we know it today.

Al-Khwarizmi's successors undertook a systematic application of arithmetic to algebra, algebra to arithmetic, both to trigonometry, algebra to the Euclidean theory of numbers, algebra to geometry, and geometry to algebra.

The Arabic mathematicians developed algebra, number theory and they made considerable contributions to geometry, trigonometry and mathematical astronomy. Astronomy, time-keeping and geography provided other motivations for geometrical and trigonometrical research. The construction of astronomical instruments such as the astrolabe was also a speciality of the Arabs.

CONCLUSIONS

Existing knowledge can pass to next generations through different codes and forms. These can be

- oral
- written
- constructions
- current technological achievements. Recently a spaceship was launched containing the basic elements of our civilization, in an effort to inform "others" about our existence.

Ancient Eastern Civilisations (mainly Hindou, Babylonians, Egyptians, Greeks) have achieved a comparatively high level of knowledge concerning philosophy, mathematics, astronomy, arts and science. This knowledge has been used to produce and construct masterpieces.

These achievements of human knowledge and creativity are often used to get some information about historical data that is missing. For example, statistical analysis together with X-ray and γ-ray spectroscopy measurements on coins found during excavations, improved our current knowledge about certain historical periods significantly.

Arabic/Islamic mathematics. Certainly many of the ideas which were previously thought to have been brilliant new conceptions due to European mathematicians of the sixteenth, seventeenth and eighteenth centuries are now known to have been developed by Arabic mathematicians almost four centuries earlier. In many respects the mathematics studied today is far closer in style to that of the Arabic contribution than to that of the Greeks.

There is a widely held view that, after a brilliant period when the Greeks laid the foundations for modern mathematics, there was a period of stagnation before the Europeans took over where the Greeks left off, at the beginning of the sixteenth century The common perception of the period of 1000 years or so between the ancient Greeks and the European Renaissance is that little happened in the world of mathematics except that some Arabic translations of Greek texts were made which preserved the Greek learning so that it was available to the Europeans at the beginning of the sixteenth century.

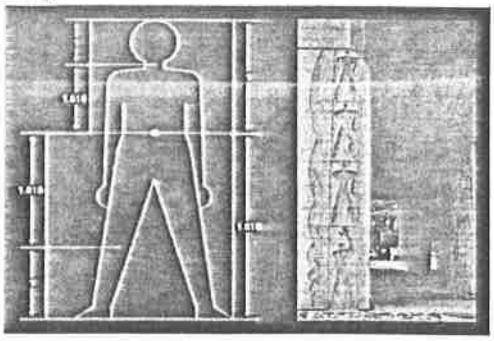
The background to the mathematical developments which began in Baghdad around 800AD is not well understood. Certainly there was an important influence which came from the Hindu mathematicians whose earlier development of the decimal system and numerals was important. There began a remarkable period of mathematical progress with al-Khwarizmi's work and the translations of Greek texts. The most important Greek mathematical texts which were translated into Arabic, are those of Euclid, Archimedes, Apollonius, Diophantus and Menelaus.

Al-Khwarizmi's major work is entitled "Al-jabr wa'lmugabalah" [15] (restoration and balancing) and from the first word in this title we now have the word "algebra". However his algebra was a rhetorical algebra which, unlike the work of Diophantus, did not use symbols for particular arithmetical operations. It is important to understand just how significant this new idea was. It was a revolutionary move away from the Greek concept of mathematics which was essentially geometry.

In contemporary art "The Sacrament of the Last Supper" by Salvador Dali (1904-1989) is painted inside a golden rectangle [13]. Golden proportions were used for positioning the figures. Part of an enormous dodecahedron floats above the table. The polyhedron consists of 12 regular pentagons and has fundamental golden connections.

Le Corbusier (1942-1948) developed a scale of proportions which he called Le Modulor [14], based on the Golden Section, using the physical dimensions of the average human. The measurement was based on a human body whose height is divided in golden section commencing at the navel. Modulor is a sequence of measurements which Le Corbusier used to achieve harmony in his architectural compositions. In many of Le Corbusier's most notable buildings, including the Chapel at Ronchamp, evidence of his Modulor system can be seen.

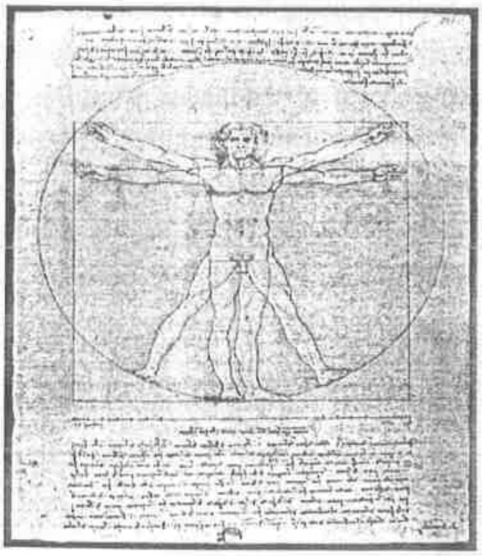
The Sacrament of the Last Supper (by S. Dali)



Le Modulor (by Le Corbusier)

Finally we ought to make a brief reference to the contribution of the Arabic mathematics to the development of the pure mathematical knowledge.

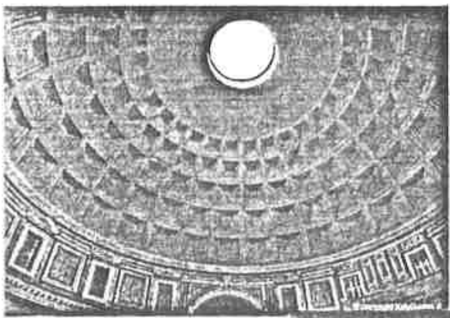
Recent research gives a new picture of the debt that we owe to over his academy door: "Let no one enter who is lacking in geometry". This phrase taken from Leonardo's notebooks gives some idea of the importance he placed on mathematics. [12]



- a palm is the width of four fingers or three inches
- a foot is the width of four palms and is 36 fingers or 12 inches
- a cubit is the width of six palms
- a man's height is four cubits and 24 palms
- a pace is four cubits or five feet
- the length of a man's outspread arms is equal to his height
- the distance from the hairline to the bottom of the chin is one-tenth of a man's height
- the distance from the top of the head to the bottom of the chin is one-eighth of a man's height
- the maximum width of the shoulders is a quarter of a man's height
- the distance from the elbow to the tip of the hand is one-fifth of a man's height
- the distance from the elbow to the armpit is one-eighth of a man's height
- the length of the hand is one-tenth of a man's height
- the distance from the bottom of the chin to the nose is one-third of the length of the head
- the distance from the hairline to the eyebrows is one-third of the length of the face
- the length of the ear is one-third of the length of the face

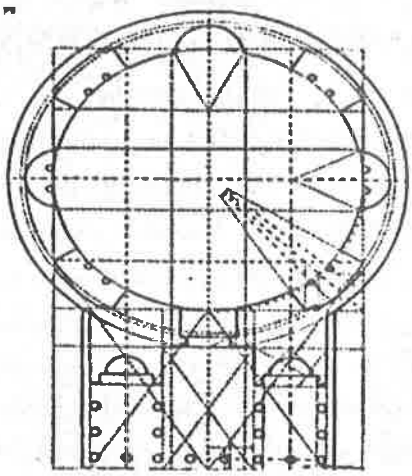
According to Augusto Marinoni, 'The problem in geometry that engrossed Leonardo interminably was the squaring of the circle". His study of nature and anatomy emerged in his realistic paintings, and his dissections of the human body paved the way for remarkably accurate figures. He was the first artist to study the physical proportions of men, women and children and to use these studies, together with the golden ratio, to present the "ideal" human figure, as it can be observed in his famous "Vitruvious man". This piece of Art is a study of the proportions of the male human body as described in a treatise by the ancient Roman architect Vitruvius.

The Pantheon



The Dome

The rectangular porch and the triangular pediment



Let us now consider some species from the art work of relatively recent years. Leonardo Da Vinci turned to science in the quest to improve his artwork. "Let no one read me who is not a mathematician." Recall Plato's inscription

The Parthenon's facade as well as elements of its facade and elsewhere can be circumscribed by golden rectangles. To the extent that classical buildings or their elements are proportioned according to the golden ratio, this might indicate that their architects were aware of the golden ratio and consciously employed it in their designs. **Alternatively, it is possible that the architects used their own sense of good proportion, and that this led to some proportions that closely approximate the golden ratio.** On the other hand, such retrospective analyses can always be questioned on the ground that the investigator chooses the points from which measurements are made or where to superimpose golden rectangles, and that these choices affect the proportions observed.

A well known building representing the genius of Roman architecture is the the Pantheon temple. The Pantheon temple to "all the gods" [*pan theon*] was originally part of Augustus Caesar's plan to rebuild Rome in his image. Agrippa originally built it as an *ovatio* ("non-military triumph") to Augustus but the philhellene emperor Hadrian transformed and finished the rebuilt monumental temple a century later around 122-24 C.E. [11].

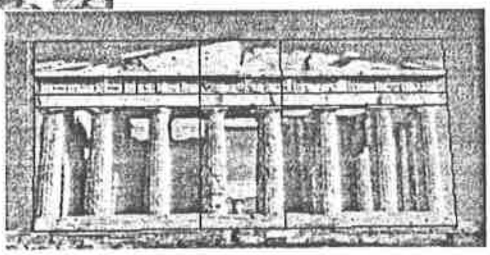
The Pantheon has three main architectural components. A rectangular porch with its triangular pediment, connected to a cylindrical drum in the main temple structure and surmounted by a dome. Thus it contains the basic Pythagorean units. Viewers can see the concept of Pythagorean geometry of cosmos in the pediment roof, rectangle of porch and hemisphere of roof that actually becomes a full sphere in that space between roof and marble floor. Some scholars interpret the architecture of Pantheon as an integrated visualization of the Pythagorean cosmos, which generate harmony, the laws of arithmetic, geometry, astronomy and musical proportions logic and methods that we still use in modern mathematics in our days.

Because of the uniqueness of the Golden ratio or the Golden section the Ancient Greeks, principally the philosophers and mathematicians, had studied it very carefully, so much so that they used it in almost all their constructions.

Plato in his studies regarding aesthetics, appears to have conceived the notion of the "Divine Unit" which is believed to give a harmonic composition to our world. He suggests that the role of numbers in combination with the sense of beauty, with harmony and rhythm, results in an organized and programmed universe [10]. Plato was influenced greatly by Pythagoras' theories which expressed ideas and arithmetic perceptions regarding analogies and universal order. The letter Phi which is used universally as the symbol of the golden section is derived from the name of the ancient Greek sculptor Pheidias. In his works, we encounter the golden section and it is just that, which creates the sense of beauty and harmony and differentiates Pheidias works from those of other great sculptors.

Some studies of the Acropolis, including the Parthenon which was created in the "classic period", conclude that many of its proportions approximate the golden ratio.

The Parthenon



A large number of examples in art, science, and nature exist, through which one can demonstrate that mankind has indeed been influenced by the nature of the irrational number pi, the golden section, and the application of mathematics, incorporating them in its culture.

Let us now point out briefly some milestones of the history of Greek civilization.

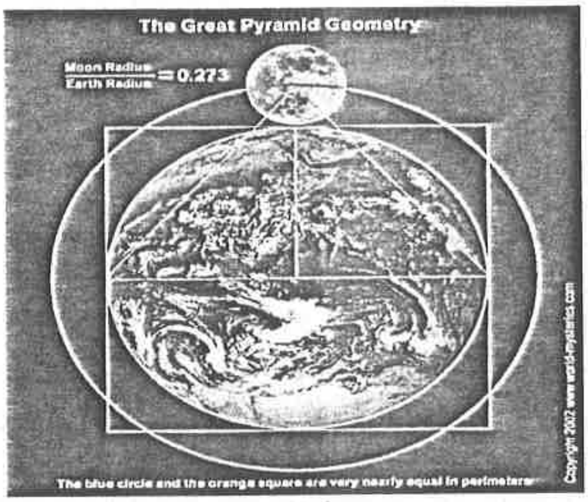
Though the Greeks certainly borrowed from other civilizations, they built a culture and civilization on their own which is impressive, influential in Western culture and decisive in founding mathematics as we know it. Greek philosophy, literature, music, sculpture, and most particularly mathematics exist as facets of the Greek heritage

The best estimate is that the Greek civilization dates back to 2800 BC just about the time of the construction of the great pyramids in Egypt.

The Greeks settled in Asia Minor, possibly their original home, in the area of modern Greece, and in southern Italy, Sicily, Crete, Rhodes, Delos and North Africa. About 775 BC they changed from hieroglyphic writing to the Phoenician alphabet. This allowed them to become more literate, or at least more facile in their ability to express conceptual thought. The ancient Greek civilization lasted until about 600 BC. Originally, the Egyptian and Babylonian influence was greatest in Miletus, a city of Ionia in Asia Minor and the birthplace of Greek philosophy, mathematics and science.

With regard to the mathematics, it is best to distinguish between the two periods: the classical period from about 600 BC to 300 BC and the Alexandrian or Hellenistic period from 300 BC to 300 A.D. Indeed, from about 350 BC until 700A.D′ the centre of mathematics moved from Athens to Alexandria (in Egypt), the city built by Ptolemy I, a Macedonian general in the army of Alexander the Great (358-323 BC) and it remained a centre of mathematics for most of the next millennium.

The detailed study of Greek mathematics reveals much about the



You might argue that all these facts are coincidental. But the coincidences are too many that they cannot be ignored. Further research is required and the scientific community has to be open-minded to any conclusion based on experimental data and scientific analysis. The issue concerning the Geometry of the Great pyramid can be concluded as follows:

- The dimensions and structure of the Great pyramid are related, within experimental errors, to numbers that are equal to pi, phi and e. This is beyond any doubt, given the fact that the scientific community has accepted, in many areas, measurements that involved significant experimental uncertainties.

- If the dimensions are accepted, the next question is: did they know? This is the domain where further research is needed. Here we deal with a civilization which was clever enough and capable of producing such monuments and much more. We have evidences that Egyptians hinted at it, at the very least.

must have had at least some mathematical understanding of the importance of these constants. By no means is it possible that these constants show up accidentally by piling up some rocks.

Squaring the circle (creating a square equivalent to a given circle in perimeter or area) was, and is, one of the unsolved problems of mathematics. According to some scolars, the square is the symbol of the Earth, the Maternity, the Matter, the four seasons, the four directions, the change. The circle, on the other hand, is the symbol of Sky, the Ether, the spirit, the internity, the periodicity, the revolution, the motion with no direction [9]. Squaring the circle therefore, was the extension of spirit into matter, the condensation of Ether, an attempt to unify Earth with Sky. It represents also, from the geometrical and mechanical point of view, the synthesis of translational and rotational motion, the helical and spiral motion.

A surprising coincidence is that, although the problem of squaring the circle is mathematically unsolved (because pi is an irrational number), the geometry of the Earth, the Moon and GP brings us as close to the solution as possible. If the base of GP is equated with the diameter of the Earth, then the radius of the moon can be generated by subtracting the radius of the Earth from the height of the pyramid.

Another coincidence is that the distance between Earth and Sun at perihelion (the time Earth is closest to the Sun) is $147x10^9$m, which is equal to $280x10^9$ royal cubits, hinting at the height of the Great Pyramid, 280 royal cubits. [7]

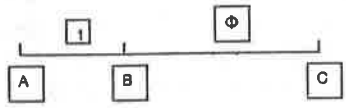
According to Euclid: "*A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less*". The extreme and mean ratio is also known as the golden ratio Phi (Φ) or golden section.

The Golden Ratio

The golden ratio can be expressed as a mathematical constant, usually denoted by the Greek letter Φ(phi). The figure of a **golden section** illustrates the geometric relationship that defines this constant. Expressed algebraically:

$$\frac{BC}{AB} = \frac{AC}{BC}$$

If AB = 1 and BC = Φ, then



$$\frac{\Phi}{1} = \frac{\Phi + 1}{\Phi}$$

$$\Phi^2 - \Phi - 1 = 0$$

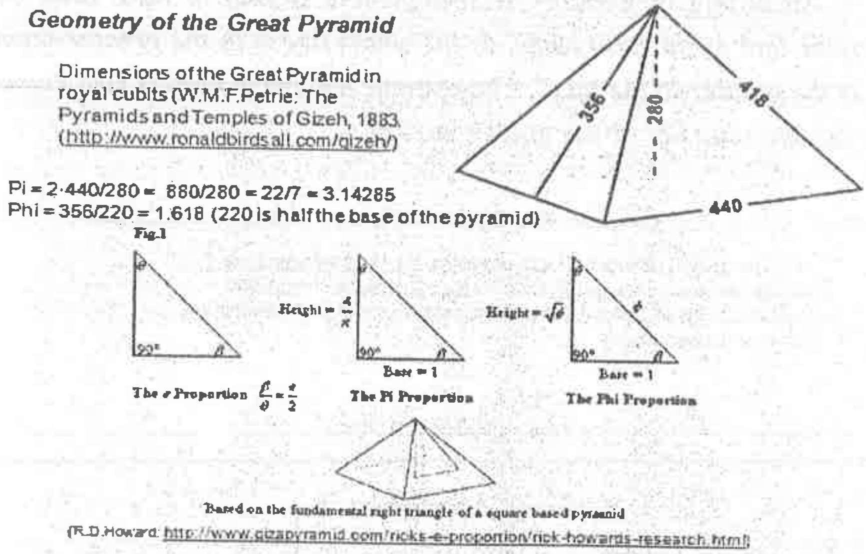
$$\Phi = \frac{1 + \sqrt{5}}{2}$$

$$\Phi = 1.61803...$$

We have left out the proof for the base of the natural logarithm e since the Triple-Triangular Theory is rather complex.

The significance of e (the base of the natural logarithms, e=2.718281828...) is very important in everyday life events. The radioactive decay law follows the equation $N=N_0e^{-\lambda t}$, the behavior of electrical R-L, R-C circuits is dictated by exponential equations, damped oscillations can be described using the exponential decay law, only to mention few of a large number of examples where the exponential factor is present.

The fact that the pyramid seems to encode Pi and Phi and e, the most profound constants in nature, shows that the builders of the pyramid



The approximate dimensions of the pyramid are measured by Petrie according to the remains of the sockets in the ground for the casing stones whose remains are still at the top of the pyramid, and the angle of the slopes. The uncertainties involved in the measurements of the dimensions of such a construction, could give rise to arguments whether or not we can use them in order to discover the existence of pi and phi into the structure of the Great Pyramid.

Pi can be found taking twice the base length of the pyramid divided by its height: pi = $2 \cdot 440/280 = 880/280 = 22/7 = 3.14285$ approximating Pi with a relative discrepancy of 0.04 % ! The engagement of pi value in the main dimensions also suggests a very accurate angle of 51°52' 2" of the slopes which expresses the value of pi.

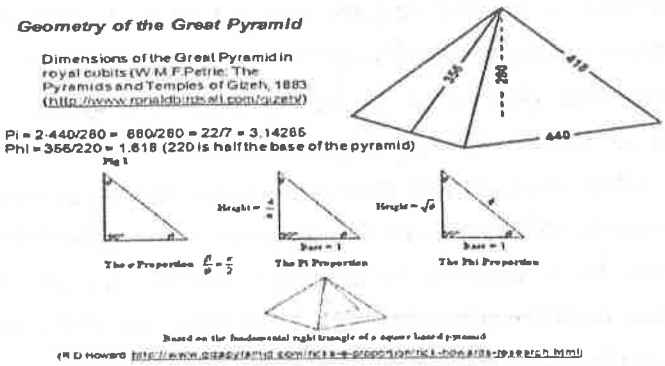
Phi (Φ) is found when the length of the sloping side of the pyramid is divided by half the base length of the pyramid, Phi = $356/220 = 1.618$ (220 is the half of the base of the pyramid), approximating Phi with a relative discrepancy of 0.01 % !!

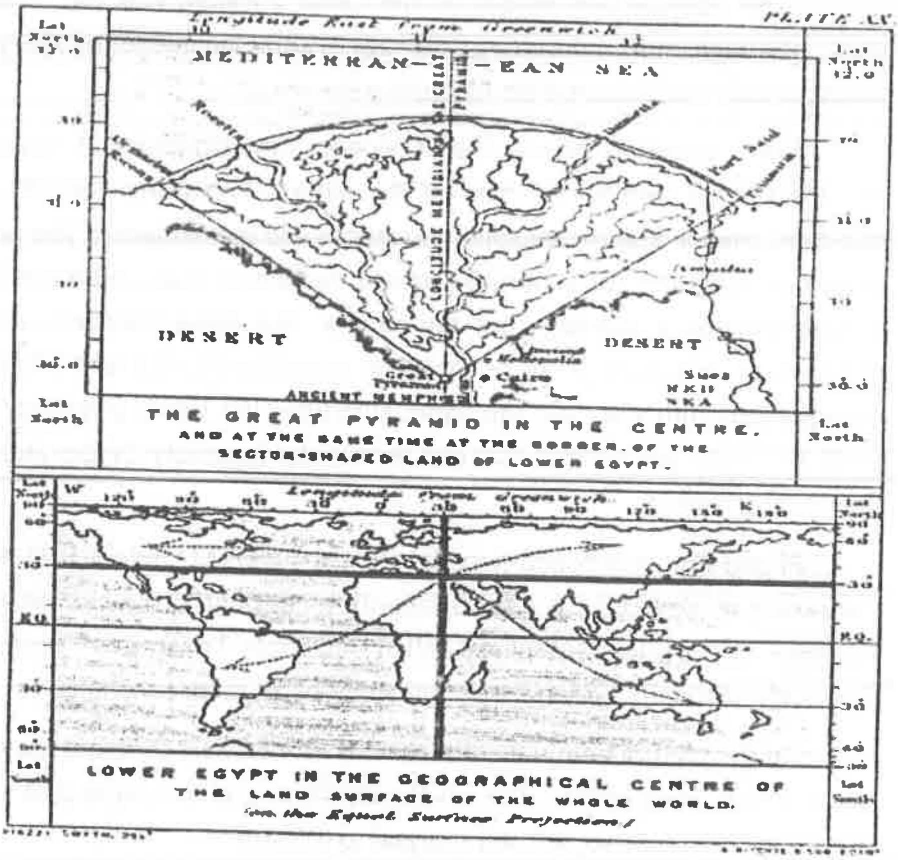
The construction details of the Great Pyramid and the techniques used, although very interesting, are not within the scope of the present paper, which will focus on the Geometry involved.

Some scientists believe that it encodes the mathematical constants Pi (π), Phi (Φ) as well as the base of the natural logarithms (e)! [7]. These constants play a significant role in Sciences and mathematics. The origin of these constants has for centuries puzzled mathematicians and scientists and it still remains a mystery, but the fact is, that these constants show up everywhere in nature. If all of the three constants Pi, Phi and e have been incorporated into one and the same structure, the Great Pyramid, this in itself is a very remarkable fact and proves the ingenuity of the builders of the Great Pyramid

Pi and Phi were found first; It took until 2003AD before R.D. Howard eventually discovered the natural logarithm constant in the geometry of the pyramid. He developed a theory called the Triple-Triangular-Theory (TTT) that mathematically underpins the presence of all three constants. [8]

In the picture below the dimensions of the Great Pyramid are shown, measured in royal cubits. The royal cubit is an ancient length unit used by the Egyptians. One royal cubit is equal to 0.524m.

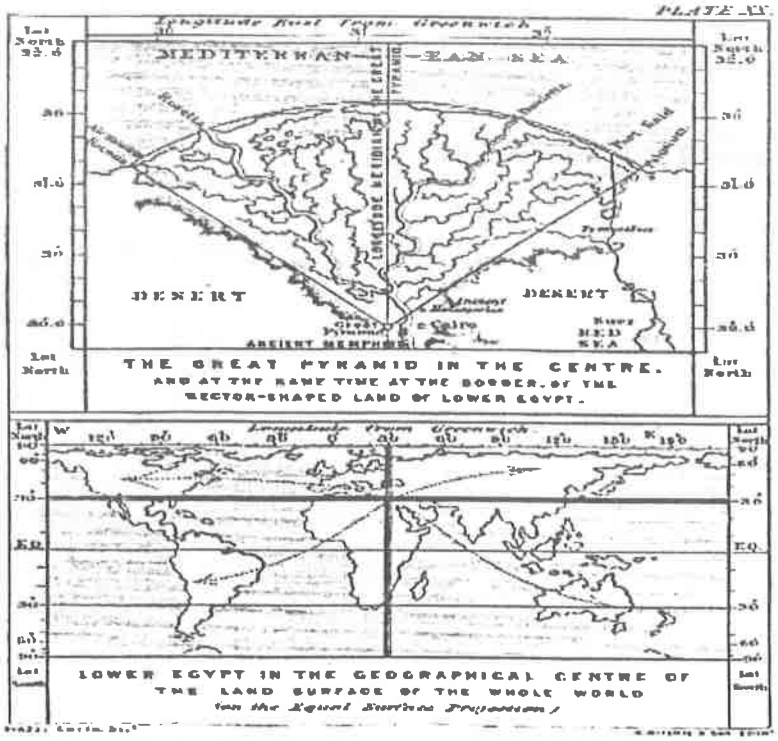




Scientists date the construction of the GP back to 2589 BC and think that it was created as a burial site, the tomb of pharaoh Cheops. In the last decade, as more of the pyramid's secrets were revealed, the tomb theory alone becomes less acceptable. In our opinion if a group of intelligent people construct such buildings they will try to incorporate the existing knowledge of their time, so that the construction will serve its purpose, but at the same time it will remain in the centuries as a channel through which knowledge can be transferred to the next generations. In any case, the civilization that built the pyramids must have had a scientific understanding and access to techniques even beyond our own.

character of its placement on the planet Earth. The Pyramid lies at the centre of mass of the continents. It also lies in the centre of all the land area of the world, dividing the earth's land mass into approximately equal quarters.

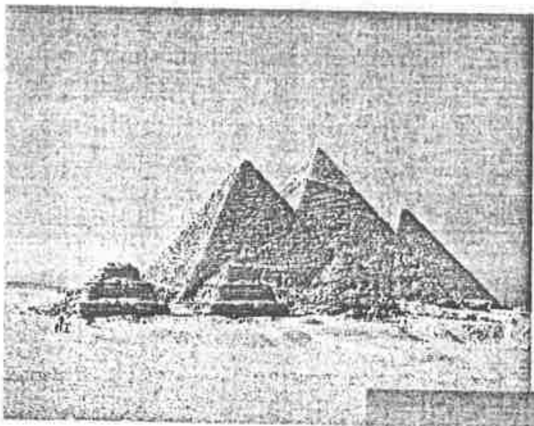
The north-south axis (31° east of Greenwich) is the longest land meridian and the east-west axis (30° north is the longest land parallel on the globe. ***The location of the Great Pyramid is the place where these longest land-lines of the terrestrial earth cross!*** [6]. Many surveys on the GP proved that the Egyptians built the sides of the pyramid along the four Cardinal points with extreme accuracy. Whether they used the stars, and/or the rising and setting Sun we don't know. Our feeling is that whatever method they used it must have been direct and simple.



of indications that the society of that period had reached a high level of achievement.

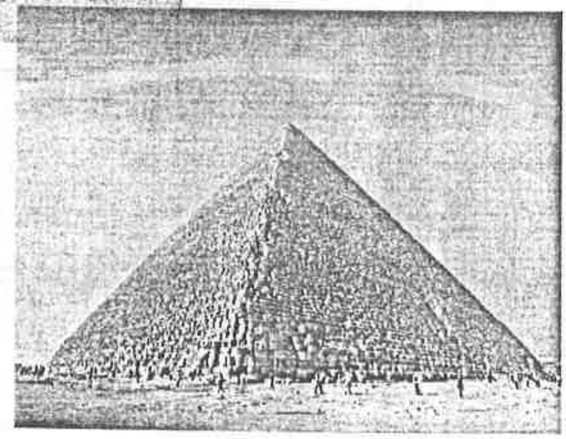
The Giza plateau contains three pyramids along with a number of satellite pyramids. These pyramids shown in the following are from left to right:

- The pyramid of Menkaure (or Mycerinos)
- The pyramid of Khafre (or Chefren), and
- The Great Pyramid of Khufu (or Cheops).



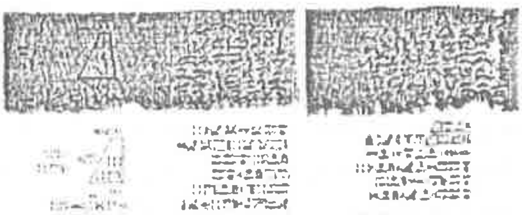
From left to right:

The pyramid of Menkaure(Mycerinos)
The pyramid of Khafre (Chefren)
The Great pyramid of Khufu (Cheops)

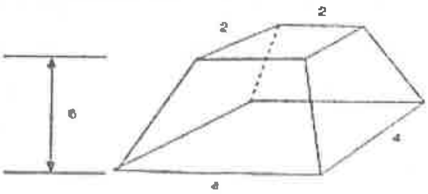


The Great pyramid of Khufu
(or Cheops)

Among the three, the Great Pyramid demonstrates the remarkable



Problem 14 states that a pyramid has been truncated in such a way that the top area is a square of length 2 units, the bottom a square of length 4 units, and the height 6 units, as shown. The volume is found to be 56 cubic units, which is correct.



The text of the example runs like this: "If you are told: a truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top: You are to square the 4; result 16. You are to double 4; result 8. You are to square this 2; result 4. You are to add the 16 and the 8 and the 4; result 28. You are to take 1/3 of 6; result 2. You are to take 28 twice; result 56. See, it is of 56. You will find (it) right"

This describes the correct calculation: $V = \frac{1}{3}6(4^2 + 4 \times 2 + 2^2)$

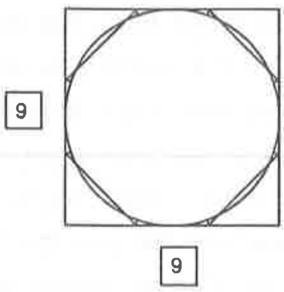
which indicates that the Egyptians knew the correct formula for obtaining the volume of a truncated pyramid:

$$V = \frac{1}{3}h(a^2 + ab + b^2).$$

One might think that the mathematical knowledge scribed on those two papyri has nothing to do with the construction of the pyramids that took place nearly one thousand years earlier. This argument has to take into account that there must have been a large number of papyri but since the material is rather fragile almost all have been destroyed. Thus we cannot exclude the possibility of the existence of papyri older than the Rhind and Moscow papyri dealing with mathematics. On the other hand early hieroglyphic numerals were found on temples, stone monuments and vases and instances of numerals written in hieratic can be found as far back as the Early Dynastic Period.

Egypt's first continuous height of civilisation in both complexity and achievement was reached during the Old Kingdom period when the pyramids were built. For example the Great Pyramid at Giza plateau by the side of Nile, one of the seven world wonders, was built around 2650 BC and it is a remarkable feat of engineering. This provides the clearest is a scroll about 6 meters long and 1/3 of a meter wide. Ahmes states that he copied the papyrus from a now-lost Middle Kingdom original, dating around 1850 BC. The work is a collection of 84 problems in arithmetic, algebra, geometry, weights and measures. On a broader level, considering the RMP and its parent documents the Egyptian fraction notation was birthed around 2,000 BC, most likely as a method to replace the Old Kingdom's binary fraction Horus-Eye notation and its awkward round off system. [4]

Problem 48. Trisect each side. Remove the corner triangles. The resulting octagonal figure approximates the circle. This gives a value of

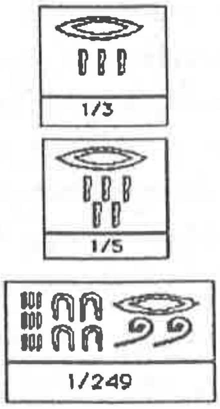


relationships using geometrical methods have always resulted in a serious and widespread artistic method".

The studies to date have proven the existence of a mathematical culture in the Arab world, especially in geometry and in numerical systems, as early as 3000 BC [2].

In Egypt, civilisation peaked very early on. Due to the development of agriculture the study of astronomy was vital in order to provide calendar information. At the same time, that is, during the Early Dynastic Period, the Egyptians had already developed their hieroglyphic writing and some elements of numerals written in hieratic form.The Egyptian number system was not well suited for arithmetical calculations, since it had similar drawbacks to that of Roman numerals. Indeed, although addition was quite satisfactory, multiplication and division were essentially impossible [3].

Hieratic numerals



Hieroglyphic numerals

We have examples of Egyptian mathematics written on the Rhind papyrus and another papyrus, the Moscow papyrus, with a translation into hieratic script.

The Rhind Mathematical Papyrus (RMP), now located in the British Museum, was scribed by **Ahmes** (c. 1680 BC-c. 1620 BC) (more accurately *Ahmose*), who lived during the Second Intermediate Period. The papyrus

Influence of the Arabic mathematics to the Western Culture through the geometry of Cheops Pyramid.

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In this presentation an effort is made to reveal how the mathematical knowledge possessed by the peoples of the Arab world influenced western culture. Although the points, opinions and examples that will be presented are based on studies emanating from various sources and do not constitute a novel concept, they justify how my personal perceptions and conclusions were formulated. Because of the broad nature of this topic, we will limit ourselves to the milestones in the development of mathematical thought in the Arab world and through eminent examples I will attempt to present this development.

In Geometry there are two widely used concepts: "size" and "relationship", which when found in proper proportion create aesthetic optical and geometrical pleasure; thus we can speak of harmonious analogies. Since the word «αρμονία/harmony» comes from the word «αρμόζω» (to become suitable or befitting) and belongs to the same family as the word «αρμόδιος» (competent or qualified), we can conclude that "harmonious relationships" are suitable relationships that befit each other [1]. That is why when we proceed to define geometric shapes and relationships we conclude that "The determination of shapes and their